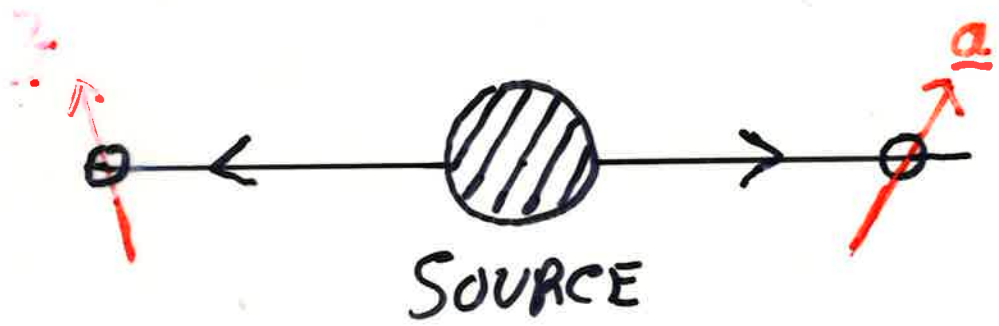
THE BELL EXPERIMENT

$\underline{a}$ and $\underline{b}$ are spin-projections parallel to $\hat{a}$ and $\hat{b}$ for two particles emitted from a common source.



Probalistic Causality

$\underline{b}$ causes $\underline{a}$

Lewis analysis (Causal Dependence)

$$\underline{b} \Box\!\!\rightarrow (Prob(\underline{a}) = p)$$

$$\neg\underline{b} \Box\!\!\rightarrow \neg(Prob(\underline{a}) = p)$$

Causal Dependence between families of events:

$$\forall \varepsilon_a \, \forall \varepsilon_b \left[(\underline{b} = \varepsilon_b) \; \Box\!\!\rightarrow \; (\mathrm{Prob}(\underline{a} = \varepsilon_a) = p^{\varepsilon_a}_{\varepsilon_b}) \right]$$

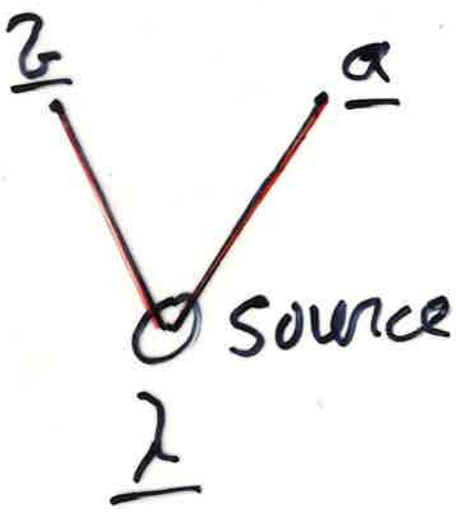
where $\forall \varepsilon_a$, $p^{\varepsilon_a}_{\varepsilon_b'} \neq p^{\varepsilon_a}_{\varepsilon_b''}$ if $\varepsilon_b' \neq \varepsilon_b''$

In the Bell experiment

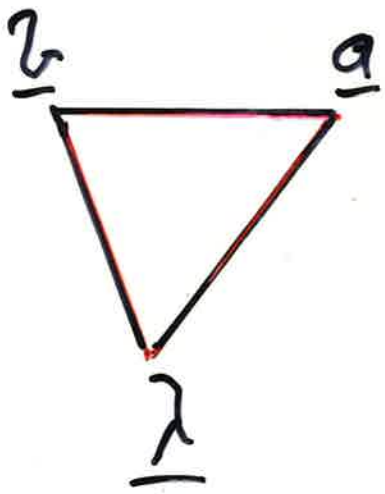
$$\left.\begin{aligned} p^{+1}_{+1} &= \sin^2 \tfrac{1}{2}\theta_{ab} \\ p^{+1}_{-1} &= \cos^2 \tfrac{1}{2}\theta_{ab} \\ p^{-1}_{+1} &= \cos^2 \tfrac{1}{2}\theta_{ab} \\ p^{-1}_{-1} &= \sin^2 \tfrac{1}{2}\theta_{ab} \end{aligned}\right\}$$

where θ_{ab} is the angle between $\hat{a}$ and $\hat{b}$.

CAUSAL EXPLANATIONS OF THE BELL CORRELATIONS



Common Cause



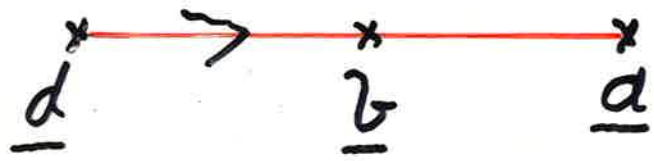
Causal 'Triangle'

Direct Cause

THE ROBUSTNESS CONDITION

Informally,

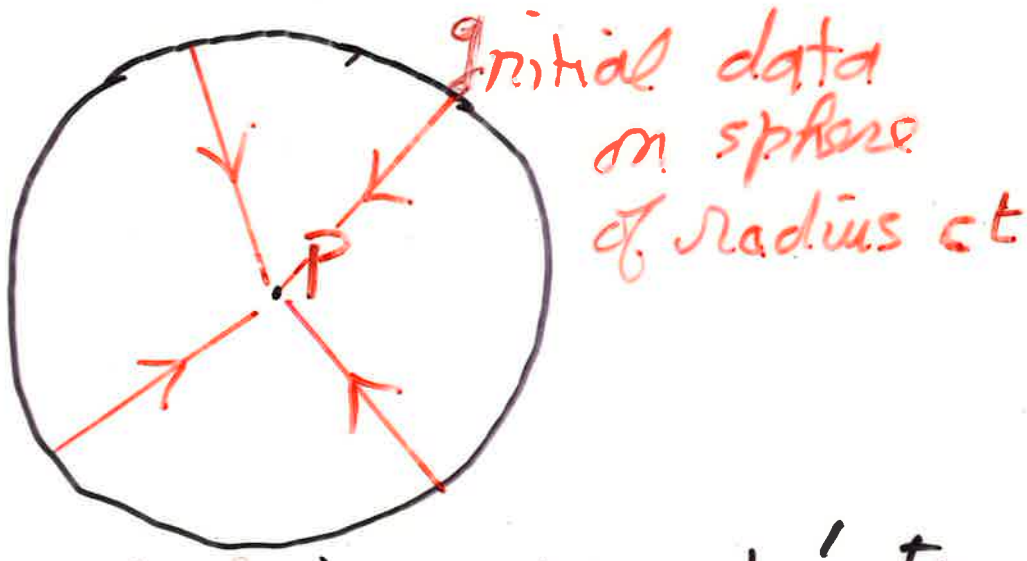
Causes produce their effects independly of how they (the causes) are brought about.



$$Prob(\underline{a} / \underline{b} \wedge \underline{d}) = Prob(\underline{a} / \underline{b})$$

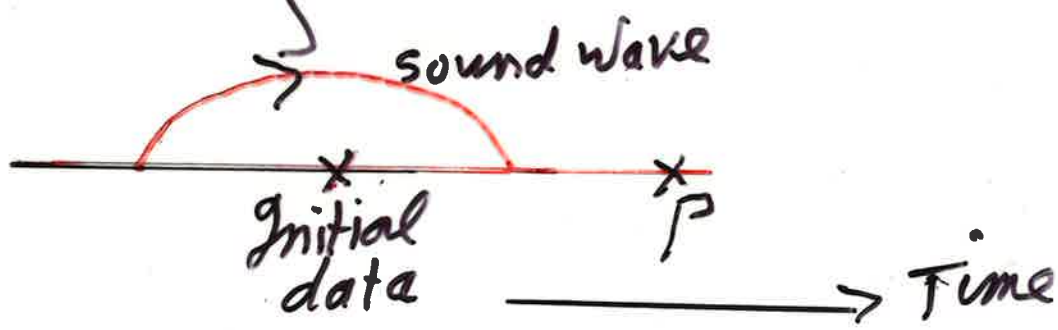
CLASSICAL PHYSICS
EXAMPLES

(1) Initial value problem for the wave equation:



Initial data 'propagate' to produce additively the total amplitude at P.

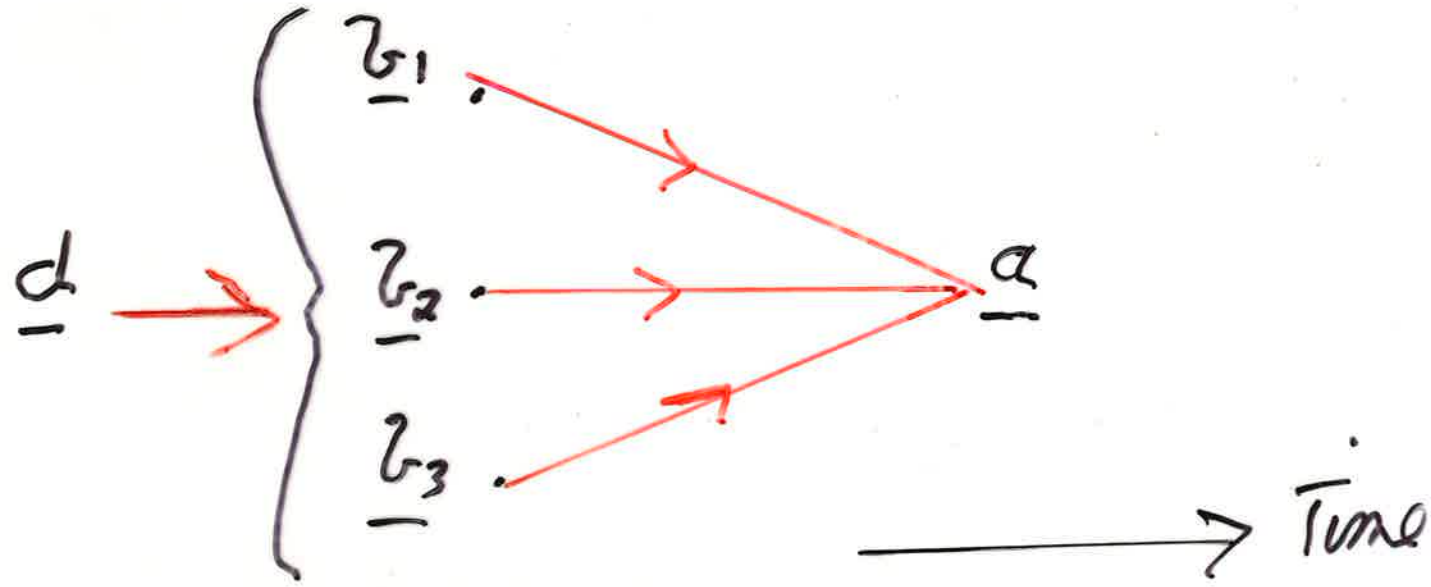
(2) Vibrating drumhead:



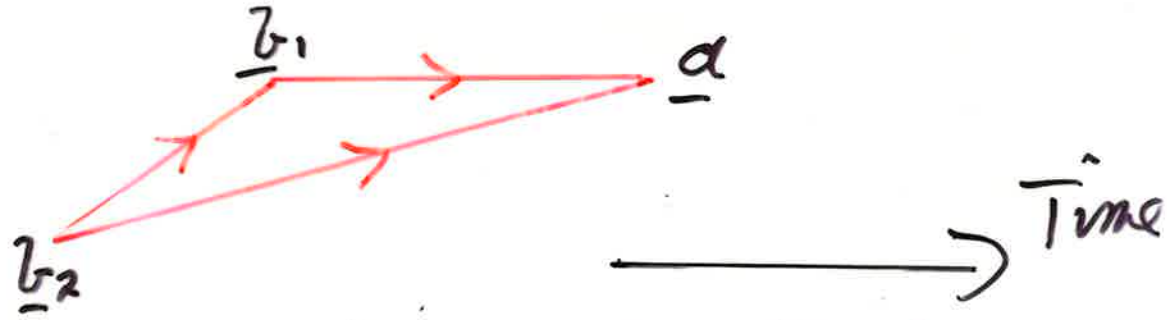
ROBUSTNESS CONDITION WITH PARTIAL CAUSES

$$Prob(\underline{a} / \underline{z}_1 \wedge \underline{z}_2 \wedge \cdots \wedge \underline{d})$$

$$= Prob(\underline{a} / \underline{z}_1 \wedge \underline{z}_2 \wedge \cdots)$$

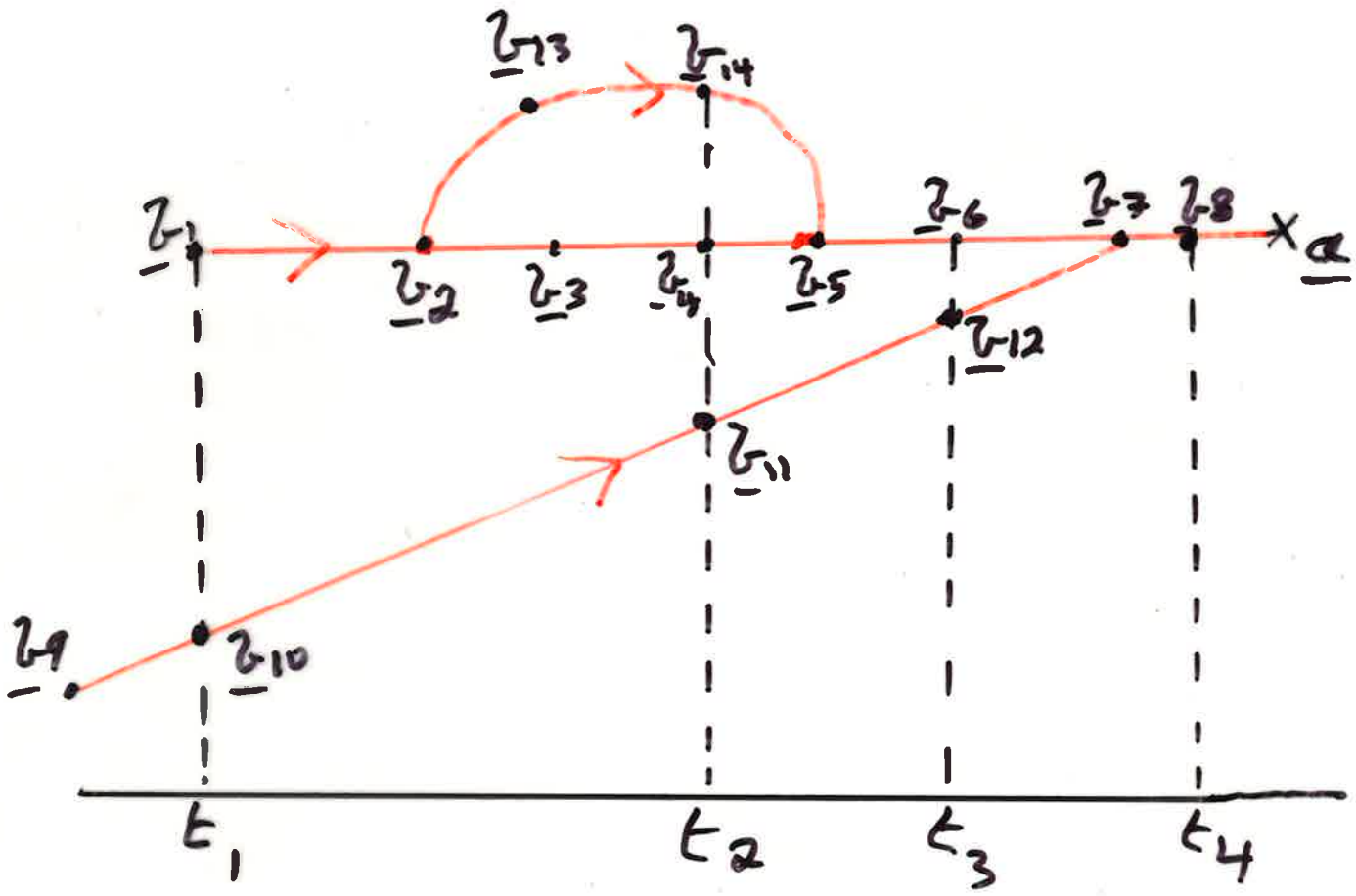


EXAMPLES



Note: $Prob(\underline{a} / \underline{z}_1)$ is not robust
counterexample to Healey's Internal Robustness

ROBUSTNESS AND MARKOV CHAINS



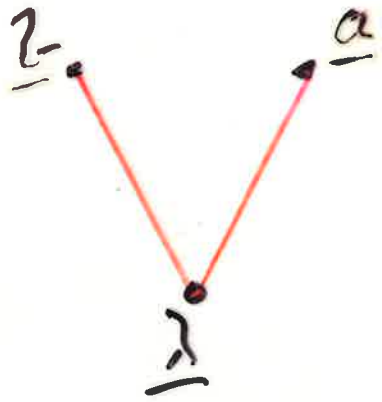
Examples of maximal sets of partial causes of $\underline{a}$:

$\{\underline{z}_4, \underline{z}_1, \underline{z}_{11}\}$

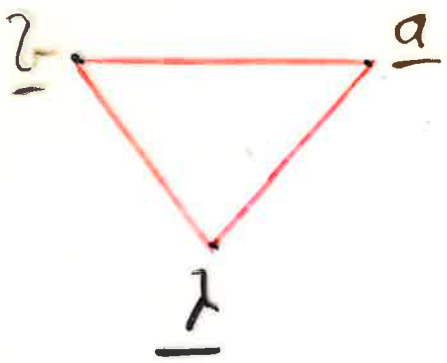
$\{\underline{z}_1, \underline{z}_9\}$
$\{\underline{z}_1, \underline{z}_{10}\}$
$\{\underline{z}_4, \underline{z}_2, \underline{z}_{11}\}$
$\{\underline{z}_{13}, \underline{z}_4, \underline{z}_{11}\}$

$\{\underline{z}_6, \underline{z}_{12}\}$
$\{\underline{z}_8\}$

QM CORRELATIONS AND ROBUSTNESS

(1) 

Ruled out by violation of Bell inequality.

(2) 

$Prob(\underline{a}/\underline{\gamma} \wedge \underline{\lambda})$ is a robust function of its arguments, so $\underline{\gamma}$ and $\underline{\lambda}$ qualify as partial causes of $\underline{a}$ (and $\underline{a}$ and $\underline{\lambda}$ of $\underline{\gamma}$!)

(3)

We can show that $Prob(\underline{a}/\underline{\gamma})$ is not robust, but should we restore robustness by considering $Prob(\underline{a}/\underline{\gamma} \wedge \Pi)$ and regard Π as a partial cause of $\underline{a}$?

THE CATALOGUE INTERPRETATION OF $\mathcal{H}$

$\mathcal{H}$ is just a catalogue of propensities $Prob(\underline{a}/\underline{a})$.

Can a catalogue be an event?

How can a catalogue be something over and above its entries, so the change in the entries is explained as 'caused' by the changes in the catalogue?

ROBUSTNESS AND SIGNALLING

$$\text{Prob}(\underline{a} = \varepsilon_a) = \sum_{\varepsilon_b} \text{Prob}(\underline{a} = \varepsilon_a / \underline{b} = \varepsilon_b) \times \text{Prob}(\underline{b} = \varepsilon_b)$$

If correlation is Robust it does not follow that changing $\text{Prob}(\underline{b} = \varepsilon_b)$ will change $\text{Prob}(\underline{a} = \varepsilon_a)$.

But we can always choose $\Delta \text{Prob}(\underline{b} = \varepsilon_b)$ so that $\Delta \text{Prob}(\underline{a} = \varepsilon_a) \neq 0$.

So No-Signalling $\longrightarrow$ Non Robustness $\longrightarrow$ No-Causality

N.B: Non Robustness is a more sensitive test of No-Causality than No-Signalling, and can be applied in situations where $\Delta \text{Prob}(\underline{b} = \varepsilon_b) = 0$.